3.8 Derivatives of inverse trig functions

Derivative of the Arcsine

$y = \sin^{-1}(x)$ means $x = \sin(y)$

$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

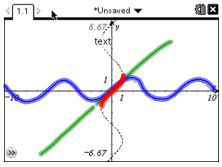
$-1 \le x \le 1$

implicit diff.

$1 = y' \cos y$

$y' = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$



restrict the range to make y=arcsine(x) a function

$\sin^2 y + \cos^2 y = 1$

$\cos y = \sqrt{1-\sin^2 y}$

$\cos y = \sqrt{1-x^2}$

Oct 4-6:55 PM

$\frac{d}{dx}\left(\sin^{-1}(x^2)\right) = 2x \cdot \frac{1}{\sqrt{1-(x^2)^2}} = \frac{2x}{\sqrt{1-x^4}}$

$\frac{d}{dx}\left(\sin^{-1}\frac{\sqrt{x}}{3}\right) = \frac{3 \cdot \frac{1}{2}x^{-\frac{1}{2}} - \sqrt{x} \cdot 0}{3^2} \cdot \frac{1}{\sqrt{1-\left(\frac{\sqrt{x}}{3}\right)^2}}$

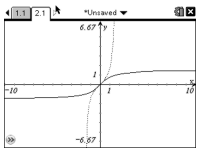
$\frac{\frac{1}{2}x^{-\frac{1}{2}}}{3}$

$\frac{1}{6\sqrt{x}} \cdot \frac{1}{\sqrt{1-\frac{x}{9}}}$

Oct 4-7:09 PM

Derivative of the Arctangent

$\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$



What is the range of y=Arctan(x) ?

Oct 4-7:14 PM

A particle moves along the x-axis so that its positon at any time $t \ge 0$ is $x(t) = \tan^{-1}\sqrt{t}$ What is the velocity of the particle when t=16?

$v = \frac{1}{2}t^{-\frac{1}{2}} \cdot \frac{1}{1+(\sqrt{t})^2}$

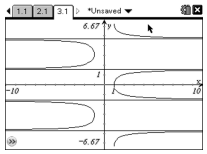
$v = \frac{1}{2\sqrt{t}} \cdot \frac{1}{1+t}$

$v(16) = \frac{1}{2 \cdot 4} \cdot \frac{1}{17} = \frac{1}{136}$

Oct 4-7:24 PM

Derivative of the Arcsecant

$\frac{d}{dx}\sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}}$



restrict the range to make y=Arcsec(x) a function

Oct 4-7:24 PM

$\frac{d}{dx}\sec^{-1}(5x^4) = 20x^3 \frac{1}{|5x^4|\sqrt{(5x^4)^2-1}}$

$= \frac{4}{x\sqrt{25x^8-1}}$

Oct 4-7:31 PM

Derivatives of the other three

$\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$

$\frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$

$\cot^{-1}x = \frac{\pi}{2} - \tan^{-1}x$

$\frac{d}{dx}\cot^{-1}x = \frac{-1}{1+x^2}$

$\csc^{-1}x = \frac{\pi}{2} - \sec^{-1}x$

$\frac{d}{dx}\csc^{-1}x = \frac{-1}{|x|\sqrt{x^2-1}}$

Oct 4-7:34 PM

Derivative of an inverse function

If *f* and *g* are inverse functions then $g'(x) = \frac{1}{f'(g(x))}$

$y = \sin^{-1}x$

$g(x) = \sin^{-1}x$

$f(x) = \sin x$

$\frac{d}{dx}\sin^{-1}x = \frac{1}{\cos y}$

Oct 4-7:38 PM